VIII. *Magnetisation of Iron.*

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[PLATES 46–52.]

Preliminary.

THE experimental determination of the relation between magnetisation and magnetising force, would be a simple matter if the expression of such relation were not complicated by the fact that the magnetisation depends not alone on the magnetising force at the instant, but also upon previous magnetising forces; in fact, if it were not complicated by the phenomena of residual magnetism. In the absence of any satisfactory theory, we can only experimentally attack particular cases, and the results obtained have only a limited application; for example, we may secure that the sample examined has never been submitted to greater magnetising force than that then operating, and we may determine a curve showing the relation of magnetisation to magnetising force when the latter is always increasing; we may also determine the residual magnetism when after each experiment the magnetising force has been removed. Such curves have been determined by ROWLAND (Phil. Mag., Aug., 1873) and others. For many purposes a more useful curve is one expressing the relation of the magnetising force and magnetisation when the former is first raised to a maximum and then let down to a defined point; such curves have been called descending curves. One or two descending curves are given in a paper by Mr. SHIDA (Proc. R. S., 1883, p. 404). It has been shown by Sir W. THOMSON and others that the magnetisation of iron depends greatly upon the mechanical force to which the iron is at the time submitted. In the following experiments the samples were not intentionally submitted to any externally applied force. CLERK MAXWELL gives in his 'Electricity and Magnetism,' chap. 6, vol. ii., a modification of WEBER'S theory of induced magnetism, and from this he deduces, amongst other things, what had been already observed, that iron may be strongly magnetised and then completely demagnetised by a reversed force, but that it will not even then be in the condition of iron which has never been magnetised, but will be more easily affected by forces in one direction than in the other. This I have verified in several cases. The ordinary determinations of residual magnetisation are not applicable to determine the permanent magnetism which a piece of the material of suitable given form will retain after removal of external magnetising force, but, as will be shown, the descending curves which express the relation of magnetisation and force, where these are diminishing, can be at once used for this purpose. Such curves can also be used, as has been shown by WARBURG and by EWING (Report Brit. Assn., 1883), to determine the energy dissipated when the magnetisation of iron is reversed between given limits. That such dissipation must occur is clear, but some knowledge of its amount is important for some of the recent practical applications of electromagnetism. Probably Professor EWING has made a more complete experimental study of magnetisation of iron than any one else. The researches of Professor HUGHES should be mentioned here, as, although his results are not given in any absolute measure, his method of experiment is remarkable beyond all others for its beautiful simplicity. I have had great doubts whether it was desirable that I should publish my own experiments at all. My reason for deciding to offer them to the Royal Society is that a considerable variety of samples have been examined, that in nearly all cases I am able to give the composition of the samples, that the samples are substantial rods forged or cast and not drawn into wire, and that determinations of specific electric resistance have been made on these rods which have some interest from a practical point of view.

Method of experiment.

Let $\mathfrak{H}$ be the magnetic force at any point, $\mathfrak{B}$ the magnetic induction, and $\mathfrak{I}$ the magnetisation (*vide* THOMSON, reprint, MAXWELL, vol. ii., 'Electricity and Magnetism'), then $\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}$. We may therefore express any results obtained as a relation between any two of these three vectors; the most natural to select are the induction and the magnetic force, as it is these which are directly observed. $\mathfrak{B}$ is subject to the solenoidal condition, and consequently it is often possible to infer approximately its value at all points, from a knowledge of its value at one, by guessing the form of the tubes of induction. $\mathfrak{H}$ is a force having a potential, and its line integral around any closed curve must be zero if no electric currents pass through such closed curve, but is equal to $4\pi c$ if c be the total current passing through the closed curve. In arranging the apparatus for my experiments, I had other objects in view than attaining to a very small probable error in individual results. I wished to apply with ordinary means very considerable magnetising forces; also to use samples in a form easily obtained; but above all to be able to measure not only changes of induction but the actual induction at any time. The general arrangement of the experiments is shown in fig. 1, and the apparatus in which the samples are placed in fig. 2. In the latter, fig. AA is a block of annealed wrought iron 457 millims. long, 165 wide, and 51 deep. A rectangular space is cut out for the magnetising coils BB. The test samples consist of two bars CC′, 12·65 millims. in diameter; these are carefully turned, and slide in holes bored in the block, an accurate but loose fit; the ends which come in contact are faced true and square; a space is left between the magnetising coils BB for the exploring coil D, which is wound upon an ivory bobbin, through the eye of which one of the rods to be tested passes. The coil D is connected to the ballistic galvanometer, and is pulled upwards by an india-rubber spring, so that when the rod C is suddenly pulled back it leaps entirely out of the field. Each of the magnetising coils B is wound with twelve layers of wire, 1·13 mm. diameter, the first four layers being separate from the outer eight, the two outer sets of eight layers are coupled parallel, and the two inner sets of four layers are in series with these and with each other. The magnetising current therefore divides between the outer and less efficient convolutions, but joins again to pass through the convolutions of smaller diameter. The effective number of convolutions in the two spools together is 2008. Referring to fig. 1, the magnetising current is generated by a battery of eight GROVE cells E, its value is adjusted by a liquid rheostat F, it then passes through a reverser G, and through a contact breaker H, where the circuit can be broken either before or at the same instant as the bar C is withdrawn; from H the current passes round the magnetising coils, and thence back through the reverser to the galvanometer K. The galvanometer K was one of those supplied by Sir W. THOMSON for electric light work, and known as the graded galvanometer, but it was fitted with a special coil to suit the work in hand. The exploring coil D was connected through a suitable key with the ballistic galvanometer L. Additional resistances M could be introduced into the circuit at pleasure, and also a shunt resistance N. With this arrangement it was possible to submit the sample to any series of magnetising forces, and at the end of the series to measure its magnetic state; for example, the current could be passed in the positive direction in the coils B, and gradually increased to a known maximum; it could then be gradually diminished by the rheostat F to a known positive value, or it could be reduced to zero; or, further, it could be reduced to zero, reversed by the reverser G, and then increased to any known negative value. At the end of the series of changes of magnetising current, the circuit is broken at H (unless the current was zero at the end of the series), and the bar C is simultaneously pulled outwards. Three successive elongations of the galvanometer L are observed. From the readings of the galvanometer K, the known number of convolutions of the coils B, and an assumed length for the sample bars, the intensity of the magnetising force $\mathfrak{H}$ is calculated. The exploring coil D had 350 convolutions. From its resistance, together with that of the galvanometer with shunts, the sensibility of the galvanometer, its time of oscillation, and its logarithmic decrement, a constant is calculated which gives the intensity of induction in the iron from the mean observed elongation of the galvanometer. The resistances have been corrected in the calculation for the error of the B.A. unit, and both galvanometers were standardised on the assumption that a certain CLARK's cell had an electromotive force of $1{\cdot}434 \times 10^8$ C.G.S. units. This CLARK's cell had been compared to and found identical with those tested by Lord RAYLEIGH.

Let the mean length of the lines of induction in the sample be l, and σ the section of the sample; let l' be the length of lines of induction in the block, and σ' their section, $\mathfrak{B}$ the intensity of induction in the sample, $\mathfrak{B}$ in the block, then $\sigma\mathfrak{B}=\sigma'\mathfrak{B}'=\mathrm{I}$; let

$$\mathfrak{B}=\mu\mathfrak{H},$$

and

$$\mathfrak{B}'=\mu'\mathfrak{H};$$

then

$$\int\left(\frac{\mathfrak{B}}{\mu}+\frac{\mathfrak{B}'}{\mu'}\right)ds=\int\mathfrak{H}ds$$

or

$$\frac{\mathrm{I}l}{\mu\sigma}=4\pi nc-\frac{\mathrm{I}l'}{\mu'\sigma'}$$

where n is the number of convolutions of the magnetising coils. Now in the instrument used σ' is large, and μ' is as large as can be obtained, hence the term $\frac{\mathrm{I}l'}{\mu'\sigma'}$ is small comparatively. My first intention was to correct the magnetising force by deducting this small correction, but finally I did not do so, because in the more interesting results the magnetism of the block is dependent in part upon previous magnetising forces, the effect of which cannot be allowed for with certainty. We know then that in all the curves the magnetising force indicated is actually too great by a small but sensible amount, which does not affect the general character of the results or their application to any practical purpose. The magnetising force then at any point of the sample is $\frac{\mathrm{I}}{\sigma\mu}=\frac{4\pi nc}{l}-$a small correction which we deliberately neglect. There is another source of uncertainty in the magnetising force: the length l is certainly greater than the space within the wrought iron block, but it is not possible to say precisely how much greater. If the sample bars and the block were a single piece, the results of Lord RAYLEIGH for the resistance of a wire soldered into a block would be fairly applicable; but it is essential that there should be sufficient freedom for the bar to slide in the hole; the minute difference between the diameters of the sample and the hole will increase the value which should be assigned to l. Throughout, l is assumed to be 32 centims., and it is not likely that this value is incorrect so much as half the radius of the bar, or 1 per cent. The magnetising forces ranged up to 240 C.G.S. units when both bars were of the same material. In some cases a single bar only was available for experiment; the plan then was to use it as the bar which enters into the exploring coil, and for the other to use a known bar of soft iron. We have then to deduct from $4\pi nc$ the magnetising force required to magnetise the bar of soft iron to the state observed, and to distribute the remainder over the shorter length of sample examined. The results obtained in this way are subject to a greater error, because some lines of induction undoubtedly make their way across from the end of the soft iron bar to the body of the block. A small correction is required, important in the case of bodies but slightly magnetic, for the fact that the area of the exploring coil is greater than the area of the bars tested. Thus the induction measured by the exploring coil is not only that in the sample, but something also in the air around the sample. The amount of this was tested by substituting for a sample of iron or steel a bar of copper, and afterwards a rod of wood, and it was found in both cases that the induction $\mathfrak{B}$ was 370 when the force $\mathfrak{H}$ was 230. The correction is in all cases small, but it has been applied in the column giving the maximum induction, as it materially affects the result when the sample contains much manganese, and is consequently very little magnetic.

The resistances were determined by the aid of a differential galvanometer, the connections being made as shown in the accompanying diagram, fig. 3. The additional resistance R was adjusted until the balance was obtained. The resistances actually measured are, some of them, as low as $\frac{1}{6000}$ of an ohm, they must not therefore be regarded as so accurate as determinations made upon samples of a more favourable form; they, however, do show the remarkable effect of several impurities in iron, though it is possible that some of the results may be in error nearly 1 per cent.

Results obtained.

In all, thirty-five distinct samples were tested, of twenty compositions. The first three were supplied to me by Messrs. Mather and Platt, and of these I have no analyses. All the rest were analysed for me in the laboratory of Sir Joseph Whitworth and Co., and the samples of material were actually prepared there, excepting, Hadfield's steel, No. X.; Bessemer iron made by the basic process for telegraph wire, No. IV., from the North-Eastern Steel Company; and two Tungsten steels, Nos. XXX. and XXXI., which are in general use for permanent magnets.

I would express here my great indebtedness to Mr. Gledhill, one of the managing directors of Sir J. Whitworth and Co., for preparing for me the samples I desired, and having them analysed. The fact is, indeed, that any value this paper may possess really lies in the variety of samples tried and in the accompanying chemical analysis, both due to Mr. Gledhill. Samples Nos. I.–X. and XXXII.–XXXV. were tested with a pair of bars, the rest with a single bar of the sample used, in combination with a bar of wrought iron. The particulars of the several samples are most conveniently given in a table which follows, and to which I shall presently refer. With many samples observations were made sufficient to plot the ascending and descending curves which express induction in terms of magnetising force, but as these can make no pretence, for reasons already stated, to such accuracy as would warrant their use in testing a theory as to the form of curves of magnetisation, a few only are given as examples, and in other cases results are given in the table sufficient to define in absolute measure the primary magnetic properties of the materials and the very characteristic way in which they differ from each other.

The curves given include in each case an ascending curve, taken before the sample had been submitted to greater magnetising forces ; a curve of residual magnetisation, that is, a curve in which the ordinate is the residual induction left after application and removal of the magnetising force represented by the abscissa, and two descending curves.

Fig. 4 gives the curves from wrought iron No. I.
Fig. 5 the same to an amplified scale of abscissæ.
Fig. 6 for steel with ·32 per cent. carbon, annealed No. VI.
Fig. 7 for steel with ·32 per cent. carbon, oil hardened No. VII.
Fig. 8 for steel with ·89 per cent. carbon, annealed No. VIII.
Fig. 9 for steel with ·89 per cent. carbon, oil hardened No. IX.
Fig. 10 for cast iron No. III.

The descending curves, which express the passage from extreme magnetisation in one direction to extreme magnetisation in the opposite direction, may be roughly defined by the maximum ordinate to which they rise, and by the points AB in which they cut the coordinate axes. The ordinate OB is what is generally meant by the residual induction after great magnetising force, or the "retentiveness." The word "Coercive Force" has been long used, but, so far as I know, in a rather vague way and without accurate definition.

I propose to call OA the "Coercive Force" of the material, and define it as that reversed magnetic force which just suffices to reduce the induction to nothing after the material has been submitted temporarily to a very great magnetising force. It is the figure which is of greatest importance in short permanent magnets. The manner in which the dimensions of the ascending curves and the curves of residual magnetisation vary with the descending curves is sufficiently obvious from inspection. The slowness with which iron or steel yields to small magnetising forces is evidently intimately connected with the coercive force. Another force is worth noting, viz., that demagnetising force which not merely reduces the induction to zero whilst applied, but just suffices to destroy the residual magnetism so that when removed no permanent magnetisation remains. The area enclosed by the two descending curves divided by 4π represents the energy dissipated when the unit volume is magnetised to saturation, its magnetism reversed, and again reversed, and so brought to its first value. This area differs a little from 4 × coercive force × maximum induction. In the cases for which curves are given the results are as follows :—

Sample.	$\frac{\text{Area from curve.}}{4\pi}$	$\frac{4 \text{ coercive force} \times \text{max. induction}}{4\pi}$
No. I.	17247	13356
„ III.	15139	13037
„ VI.	45903	40120
„ VII.	61898	65786
„ VIII.	50521	42366
„ IX.	74371	99401

In this we note that for soft iron the area is greater than the product, the reverse for hard steel; for any practical purpose we may assume that the greatest dissipation of energy which can be caused by a complete reversal to and fro of magnetisation is approximately measured by $\frac{\text{coercive force} \times \text{maximum induction}}{\pi}$. An interesting feature in the curves is the manner in which the residual magnetism rapidly attains to near its maximum value, and is then nearly constant, whilst the induction continues to increase. This is very marked in the case of cast iron.

The column of figures in the general table of results almost explain themselves.

In the case of the cast iron, the total and the graphitic carbon are given, the difference being the combined carbon. In the case of the manganese steel and iron, the induction is almost proportional to the magnetising force, hence permeability is really the magnetic property to be noted: this is given below in a separate table. The demagnetising force is that reverse force which, when applied after great magnetising force, just suffices to remove all permanent magnetisation. The energy dissipated is $\frac{\text{OA} \times \text{maximum induction}}{\pi}$, and is approximately the energy in ergs. converted into heat in a complete cycle of magnetisation from the limit in one direction to that in the opposite and back again.

In the general table of results one of the striking features is the high specific resistance of some samples of cast iron, ten times as great as wrought iron. This fact is not without practical importance in some forms of dynamo machines, for the energy wasted by local currents induced in the iron by given variations of the magnetic force will be but $\frac{1}{10}$th as great with cast iron as with wrought iron. The high resistance of cast iron may be due in large measure to its heterogeneity; grey cast iron may be regarded as a mechanical mixture of more or less pure iron with very small bits of graphite.

[Jan. 15, 1886.—I have recently determined the rate of variation with temperature of the electric resistance of a sample of cast iron for the purpose of ascertaining whether it approximated more nearly to a pure metal, to an alloy, or to bodies the resistance of which decreases with rise of temperature. The sample examined was a thin rod of grey iron 6·71 millims. diameter and 24·85 centims. long between the contacts. The range of temperature was 10° C. to 130° C., and through this range the rate of increase of resistance was nearly uniform. The specific resistance at 0° C. was inferred to be 0·000102, and the rate of increase was 0·00083 per degree centigrade.—J. H.]

Another very striking feature is the way in which any substantial proportion of manganese annihilates the magnetic property of iron; the sample with 12 per cent. of manganese is practically non-magnetic. The induction noted in the table =310 corresponds to a magnetising force of 244. If all the substances in this sample other than the iron were mechanically mixed with the iron, and arranged in such wise as to have the greatest effect upon its magnetic property, no such annihilation of magnetic property would ensue. This question of mixture will be considered somewhat more closely below.

The permeability and susceptibility are given in the following table for some of the samples containing much manganese :—

No.	Permeability.	Susceptibility.
X.	1·27	·0215
XIV.	3·59	·206
XVI.	3·57	·2046
XXXV.	1·84	·0668

It is therefore clear that the small quantity of manganese present enters into that which must be regarded for magnetic purposes as the molecule of iron, and completely changes its properties. The fact is one which must have great significance in any theory as to what is the molecular nature of magnetisation. Another clearly marked fact is the exceptionally great effect which hardening has both upon the magnetic properties and the electrical resistance of chrome steel.

Note also that in those cases where the maximum induction is low, the residual magnetism is proportionately lower still, but that the coercive force is not uniformly lower. This is in accordance with the supposition that these samples are to be regarded as mechanical mixtures of a strongly magnetic substance, such as ordinary iron or steel, and a non-magnetic substance, such as manganese steel with 12 per cent. of manganese. A feature present in all the curves may some day have a bearing on the molecular theory of magnetism. It is this : the ascending curve twice crosses the continuation of the descending curve; in other words, the fact that a sample has been strongly magnetised in a reverse direction, renders it for small forces, or for large forces, more difficult to magnetise than a virgin sample, but distinctly easier for intermediate forces. This is best seen in the case of the hardest steel, No. IX., fig. 9, in which the two curves cut in the points marked M, N. A similar phenomenon has been observed and investigated by G. WIEDEMANN (*vide* 'Die Lehre vom Galvanismus,' first edition, vol. ii., p. 340, *et seq.*).

No.	Description.	Temper.	Chemical analysis.						Specific resistance.	Magnetic properties.*				Energy dissipated.
			Total carbon.	Manganese.	Sulphur.	Silicon.	Phosphorus.			Maximum induction.	OB, residual induction.	OA, coercive force.	Demagnetising force.	
I.	Wrought iron	Annealed	..	..	..	..	..	..	·00001378	18,251	7,248	2·30	..	13,356
II.	Malleable cast iron	,,	..	..	..	..	..	..	·00003254	12,408	7,479	8·80	..	34,742
III.	Grey cast iron	..	..	..	..	..	..	..	·00010560	10,783	3,928	3·80	..	13,037
IV.	**Bessemer** mild steel	..	·045	·200	·030	None	·040	..	·00001050	18,196	7,860	2·96	..	17,137
V.	**Whitworth** mild steel	Annealed	·090	·153	·016	,,	·042	..	·00001080	19,840	7,080	1·63	..	10,289
VI.	,, ,,	,,	·320	·438	·017	·042	·035	..	·00001446	18,736	9,840	6·73	..	40,120
VII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00001390	18,796	11,040	11·00	..	65,786
VIII.	,, ,,	Annealed	·890	·165	·005	·081	·019	..	·00001559	16,120	10,740	8·26	..	42,366
IX.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00001695	16,120	8,736	19·38	..	99,401
X.	**Hadfield**'s manganese steel	..	1·005	12·360	·038	·204	·070	..	·00006554	310	..	..	..	..
XI.	Manganese steel	As forged	·674	4·730	·023	·608	·078	..	·00005368	4,623	2,202	23·50	37·13	34,567
XII.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00003928	10,578	5,848	33·86	46·10	113,963
XIII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00005556	4,769	2,158	27·64	40·29	41,941
XIV.	,, ,,	As forged	1·298	8·740	·024	·094	·072	..	·00006993	747	..	..	..	..
XV.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00006316	1,985	540	24·50	50·39	15,474
XVI.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00007066	733	..	..	..	..
XVII.	Silicon steel	As forged	·685	·694	,,	3·438	·133	..	·00006163	15,148	11,073	9·49	12·60	45,740
XVIII.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00006185	14,701	8,149	7·80	10·74	36,485
XIX.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00006195	14,696	8,084	12·75	17·14	59,619
XX.	Chrome steel	As forged	·532	·393	·020	·220	·041	Chromium. ·621	·00002016	15,778	9,318	12·24	13·87	61,439
XXI.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00001942	14,848	7,570	8·98	12·24	42,425
XXII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	,,	·00002708	13,960	8,595	38·15	48·45	169,455
XXIII.	,, ,,	As forged	·687	·028	,,	·134	·043	1·195	·00001791	14,680	7,568	18·40	22·03	85,944
XXIV.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00001849	13,233	6,489	15·40	19·79	64,842
XXV.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	,,	·00003035	12,868	7,891	40·80	56·70	167,050
XXVI.	Tungsten steel	As forged	1·357	·036	None	·043	·047	Tungsten. 4·649	·00002249	15,718	10,144	15·71	17·75	78,568
XXVII.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00002250	16,498	11,008	15·30	16·93	80,315
XXVIII.	,, ,,	Hardened cold water	,,	,,	,,	,,	,,	,,	·00002274	..	..	..	..	..
XXIX.	,, ,,	Hardened tepid water	,,	,,	,,	,,	,,	,,	·00002249	15,610	9,482	30·10	34·70	149,500
XXX.	,, ,, (French)	Oil hardened	·511	·625	None	·021	·028	3·444	·00003604	14,480	8,643	47·07	64·46	216,864
XXXI.	,, ,,	Very hard	·855	·312	..	·151	·089	2·353	·00004427	12,133	6,818	51·20	70·69	197,660
XXXII.	Grey cast iron	..	3·455	·173	·042	2·044	·151	Graphitic carbon. 2·064	·00011400	9,148	3,161	13·67	17·03	39,789
XXXIII.	Mottled cast iron	..	2·581	·610	·105	1·476	·435	1·477	·00006286	10,546	5,108	12·24	..	41,072
XXXIV.	White cast iron	..	2·036	·386	·467	·764	·458	None	·00005661	9,342	5,554	12·24	20·40	36,383
XXXV.	Spiegeleisen	..	4·510	7·970	Trace	·502	·128	..	·00010520	385	77	..	..	..

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* The maximum magnetisation is obtained from the maximum induction by subtracting 240 (the magnetising force used) and dividing by 4π, or in most cases sufficiently nearly by multiplying by 0·08. The residual magnetism is obtained by dividing the residual induction by 4π. The numbers in the table are the actual magnetic forces in the infinitesimal gap between the two bars.

Magnetisation of a mixture of magnetic and non-magnetic substance.

We suppose that the mixture is purely mechanical, and that the two substances each retain their magnetic properties.

We may regard as an element of the substances a portion great in comparison with the size of the pieces of the two substances constituting the mixture, or we may be more analytical and regard as an element a portion very small in comparison with such pieces.

Let the volume of magnetic substance be λ, of non-magnetic $1-\lambda$. The magnetic properties of the mixture will depend, not only upon λ, but upon the relative arrangement of the magnetic and non-magnetic parts.

Let α, a, A be the magnetic force, induction and magnetisation, regarding the sizes of the parts of the two substances as infinitely small; let α_0, a_0, A_0 be their values within a portion of magnetic substance. α, a, A are what we could actually observe. The relations of α_0, a_0, A, may be known from experiments on the magnetic substance when unmixed.

1. Suppose the magnetic substance to be arranged in the mixture in the form of filaments or laminæ parallel to the lines of magnetic force, then $\alpha=\alpha_0$, and $A=\lambda A_0$. Hence the effect of admixture in this case is to reduce the magnetisation for a given force in the ratio $1:\lambda$.

2. Let the non-magnetic substance be in thin laminæ lying perpendicular to the lines of force; we shall then have again $A=\lambda A_0$; but $a=a_0$ instead of $\alpha=\alpha_0$, whence

$$\begin{aligned}\alpha&=a-4\pi A\\&=a_0-4\pi\lambda A_0\\&=(1-\lambda)a_0+\lambda\alpha_0\end{aligned}$$

α_0 being supposed known in terms of a_0, this gives us the means of calculating the properties of the mixture.

These two are the extreme cases; all other arrangements of the two substances will have intermediate effects approximating to the one extreme or the other in a manner which we can judge in a rough way.

For example, if the magnetic substance be in separate portions bedded in the non-magnetic substance, the result will be somewhat analogous to the case of plates perpendicular to the lines of force; if on the other hand the non-magnetic substance be in separate portions bedded in the magnetic, the result will approximate rather to the case of filaments parallel to the lines of force.

Suppose that in the case of HADFIELD'S steel, No. X., the mixture be of pure iron in very small quantity in a non-magnetic matrix, how much pure iron is it necessary to suppose to be present, supposing the arrangement to be as unfavourable as possible? Here $a=a_0=310$, $\alpha=244$, $\alpha_0=$sensibly zero, whence $\lambda=\frac{66}{310}=0{\cdot}21$. Suppose, however, that the iron were arranged as small spheres bedded in the non-magnetic substance, we have $\lambda = -\frac{1}{2} + \frac{3}{2}\frac{\mu}{2+\mu}$, μ being the observed value of $\frac{a}{\alpha}$, viz., 1·27 ; whence $\lambda = 0{\cdot}09$. We may say that of the 86 per cent. iron in this sample not more than 9 per cent. is magnetic.

If hard steel were bedded as small particles in a non-magnetic matrix, we should expect the mixture to have low retentiveness, but comparatively high coercive force, such as we see in the case of samples XI., XIII., and XV. If our apparatus had been sufficiently delicate to detect residual magnetism in samples X., XIV., and XVI., it is probable enough that we should have found the coercive force to be considerable.

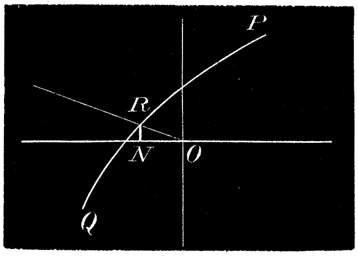
In the case of mixtures much will depend on the relative fusibility of the magnetic and non-magnetic substances. If the former were less fusible, it would probably occur as crystals separated from each other by a non-magnetic matrix ; if on the other hand it were more fusible, it would remain continuous. It is easy to see the kind of difference in magnetic property which would result.

Determination of permanent magnetisation of an ellipsoid.

If an ellipsoid be placed in a uniform magnetic field, its magnetisation will be uniform.

If the externally applied magnetising force be zero, the force at any point within the ellipsoid will be AL, BM, CN, where A, B, C are the components of magnetisation of the ellipsoid, and $L = 4\pi abc\frac{d\phi_0}{da^2}$, &c., where $\phi_0 = \int_0^\infty \frac{d(\phi^2)}{\sqrt{(a^2+\phi^2)(b^2+\phi^2)(c^2+\phi^2)}}$, and a, b, c are the semi-axes of the ellipsoid. Suppose the forces have all been parallel to the axis a, we have then $\mathfrak{H} = L\mathfrak{I} = \frac{L\mathfrak{B}}{4\pi}$ very nearly.

Let the curve PQ be the descending curve of magnetisation (the ordinates being induction), draw OR so that $\frac{RN}{ON} = \frac{4\pi}{L}$; then RN is clearly the induction in the ellipsoid when the external force is removed. In the case of a sphere $L = -\frac{4}{3}\pi$, therefore $RN = -3ON$. The greatest residual induction which a sphere of the materials can



retain is a very little less than three times the force required to reduce the magnetisation to zero.

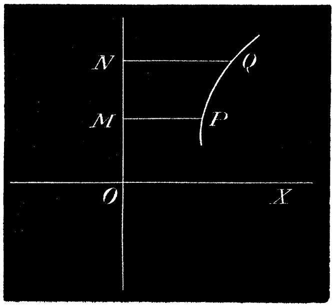
In a similar way any spheroid could be readily dealt with, and the best material judged for a permanent magnet of given proportions. It should, however, be noted that any conclusions thus deduced might be practically vitiated by the effect of mechanical vibration in shaking out the magnetism from the magnet.

Dissipation of energy by residual magnetism.

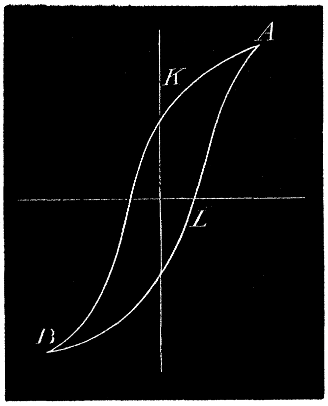
Imagine a conducting circuit of resistance R, let x be the current in it at time t, E the electromotive force other than that due to the electro-magnetic field, and a the total magnetic induction through the circuit, then

$$\mathrm{R}x=\mathrm{E}-\frac{da}{dt}.$$

The work done in time dt by the electromotive force is $x\mathrm{E}dt=(\mathrm{R}x^2+x\frac{da}{dt})dt$; of this $\mathrm{R}x^2dt$ goes to heat the wire, the remainder, or xda, goes into the electro-magnetic field. Imagine a surface of which the conducting circuit is a boundary, and on it take an elementary area; through this area draw a tube of induction returning into itself; the line integral of force along the closed tube is $4\pi x$. If therefore we assume that the work done in any elementary volume of the field is equal to that volume multiplied by the scalar of the product of the change of induction, and the magnetising force divided by 4π, the assumption will be consistent with the work we know is done by the electromotive force E. Now apply this to any curve connecting induction and magnetic force. Let PQ be two points in the curve, draw PM and QN parallel to the axis of magnetic force OX; the work done on the field per cubic centimeter passing from P to Q is equal to $\frac{\text{area PMNQ}}{4\pi}$. Some of this is converted into heat in the case of iron, for we cannot pass back from Q to P by diminishing the magnetising force.



Let AKB be the curve connecting A and B when the magnetising force is reversed, BLA when it is again reversed in this cycle; the final magnetisation is the same as it was initially; hence the balance of work done upon the field must be converted into heat; this heat will be represented by the area AKBLA $\div 4\pi$ in ergs. per cubic centimeter.



An approximation to the values of this dissipation is given in the table of results. It may be worth while to call attention to their practical application. Take the case of a dynamo-machine with an iron core, finely divided to avoid local electric currents. Note that we are going to assume—though whether true or false we do not know—that the dissipation is the same whether the magnetisation is reversed by diminishing and increasing the intensity of magnetisation without altering its direction, or whether it is reversed by turning round its direction without reducing its amount to zero.

A particular machine has in its core about 9000 cubic centims. of soft iron plates; the resistance of its armature is 0·01 ohm, of its shunt magnets 8·0 ohms, and when running 900 revolutions per minute, its E.M.F. at the brushes is 55 volts. When the current in the armature is 250 ampères we have

	Ergs. per second.
Total energy of current	$= 144 \times 10^9$.
Loss in armature resistance	$= 625 \times 10^7$.
Loss in magnet resistance	$= 378 \times 10^7$.
Loss in magnetising and demagnetising iron core of armature	= 9000 cubic centims. × 15 revolutions per second × 13,356 (from table of results) $= 18 \times 10^8$.

From this we see at once that the heat generated in the core of the armature by reversal of magnetisation is about one-half of that arising from the resistance of the copper wire of the electro-magnet. If a hard steel were used the loss from reversal might amount to 20 per cent. or more of the useful work done.

WEBER'S *Theory of Magnetism.*

In WEBER'S theory it is, in effect, assumed that the magnetic force tending to deflect a molecule is that which it would experience if it were placed in a long cylindrical cavity, the axis of the cylinder being in the direction of magnetisation. This seems a rather unnatural supposition. If instead of this we assume that the deflecting force is that which it would experience in a spherical cavity, and draw a curve connecting either the induction or magnetisation with the deflecting force on a molecule within a spherical cavity, we shall find that the curve differs very little from a straight line. In the curves already given we have taken $\mathfrak{B}$ and $\mathfrak{H}$ as the variables, where

$$\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}.$$

Suppose we take $\mathfrak{B}$ and $\mathfrak{K}$ where

$$\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}.$$

and

$$\begin{aligned}\mathfrak{K}&=\mathfrak{H}+\frac{4\pi}{3}\mathfrak{I}.\\&=\tfrac{2}{3}\mathfrak{H}+\tfrac{1}{3}\mathfrak{B}.\end{aligned}$$

The curves would then be hardly distinguishable from straight lines, the same scales being used for ordinates and abscissæ; it requires no great stretch of imagination to suppose that if this curve were continued far enough it would differ but little from that given by MAXWELL, vol. ii., p. 79.

Now, in dealing with WEBER'S theory it would seem more suitable to take $\mathfrak{K}$, the magnetic force in a spherical cavity, as the independent variable. If we assume WEBER'S theory with this modification we arrive at the following conclusions:—

1. All observations yet made upon the magnetisation of iron are upon the straight part of WEBER'S curve.

2. The particular features of curves of magnetisation as ordinarily observed arise from a slight irregularity in WEBER'S curve, magnified by the near approach of iron to a state in which a random distribution of the magnetic axes of the molecules is unstable.

I do not put these remarks forward as indicating more than the fact that we are a very long way from obtaining a range of facts sufficiently extended for testing a molecular theory of magnetism. The broad fact which strikes the mind most forcibly is the specific difference which exists between magnetic and non-magnetic bodies. Most bodies are either very slightly ferro-magnetic or very slightly diamagnetic. On the other hand iron, nickel, and cobalt are enormously magnetic.

Iron with 12 per cent. of manganese, and some small quantities of carbon and other substances, is so little magnetic that its magnetism would be accounted for by supposing that in its mass were distributed a few little bits of pure iron. There seems to be a certain instability of something we know not what; bodies fall on one side practically non-magnetic, on the other enormously magnetic, but hardly any intermediate class exists.

The number of actual observations made on each of the samples named has been very considerable, though I have not thought it necessary to set them out at length, as I base no general conclusion upon them. The bulk of these observations were made by my assistant, Mr. E. TALBOT, and my pupil, Mr. PAUL DIMIER, to whom my thanks are due for their patience and care.

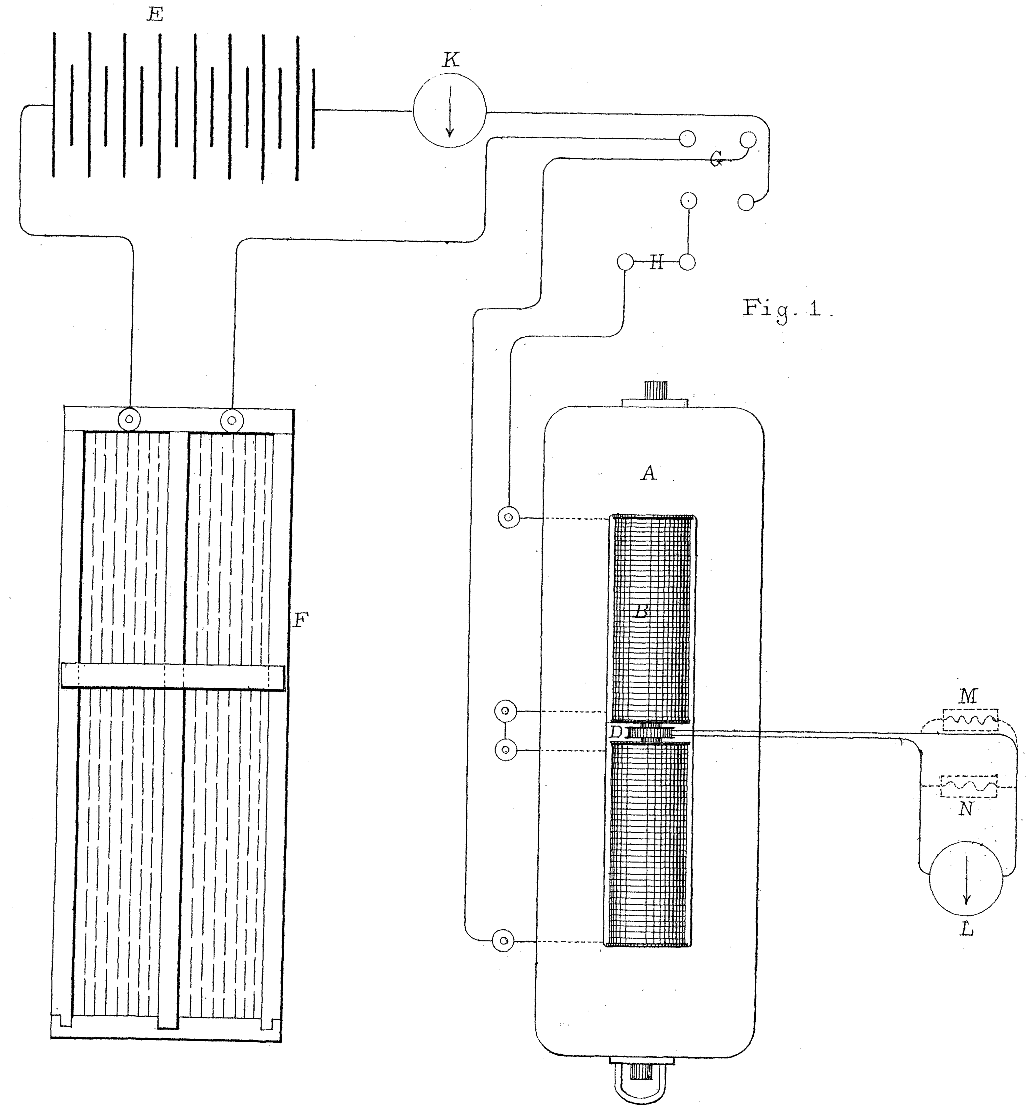


Fig. 1.

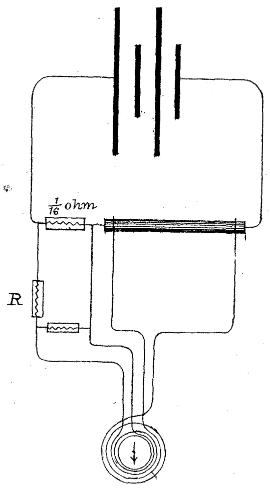
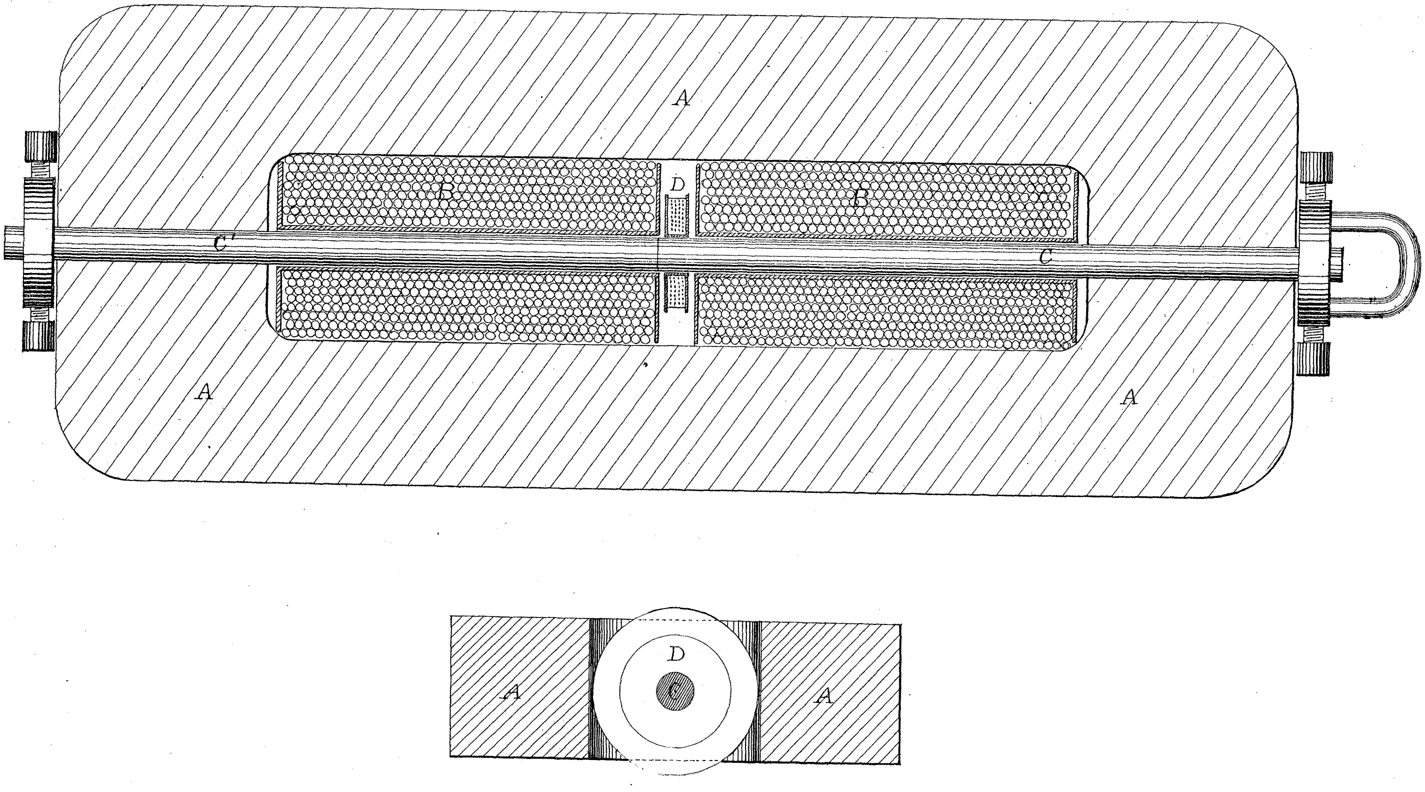


Fig. 3.

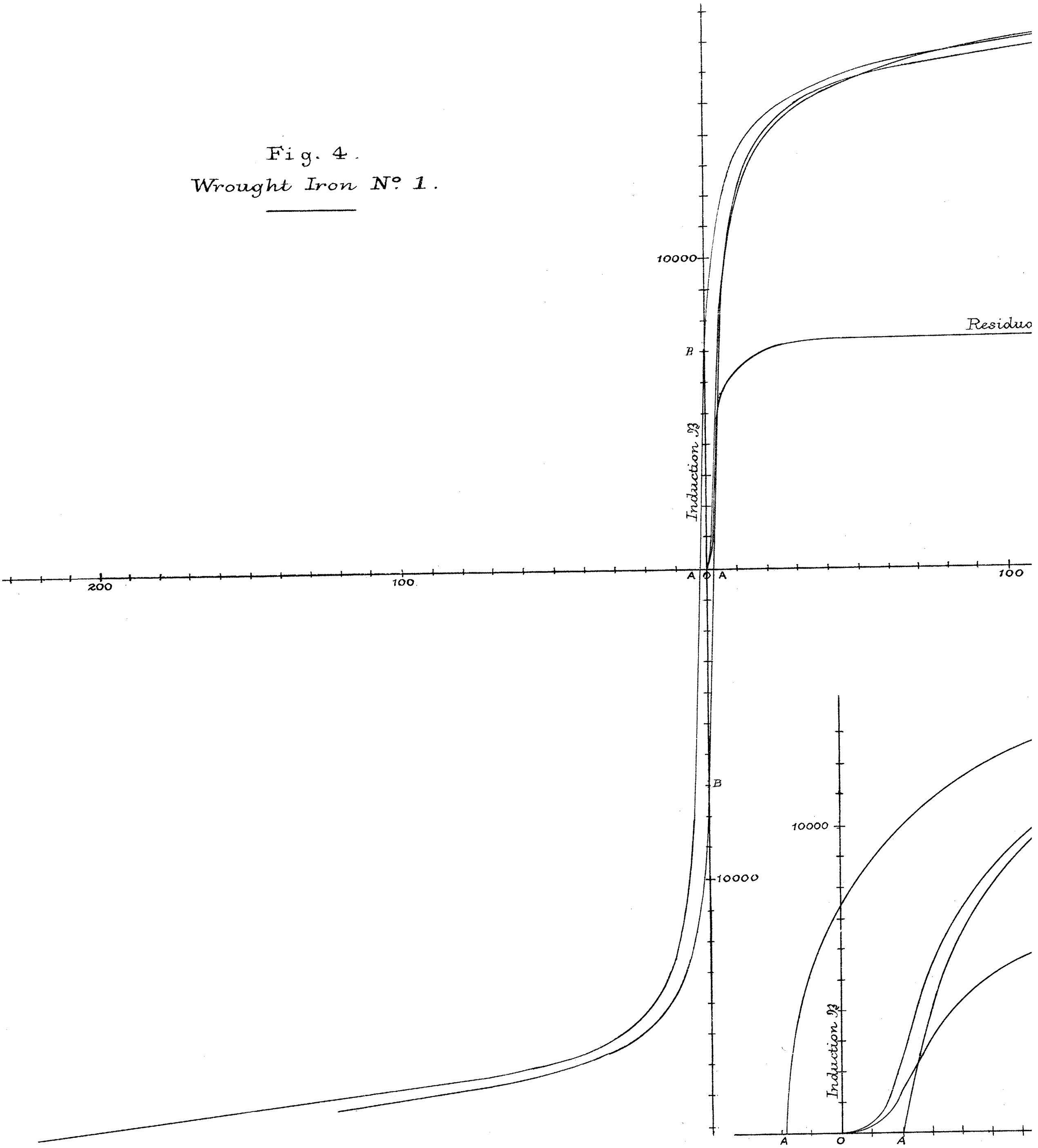
Fig. 2.



West, Newman & Co. sc.

Fig. 4.

Wrought Iron N°. 1.



West, Newman & C°. lith.

Hopkinson.

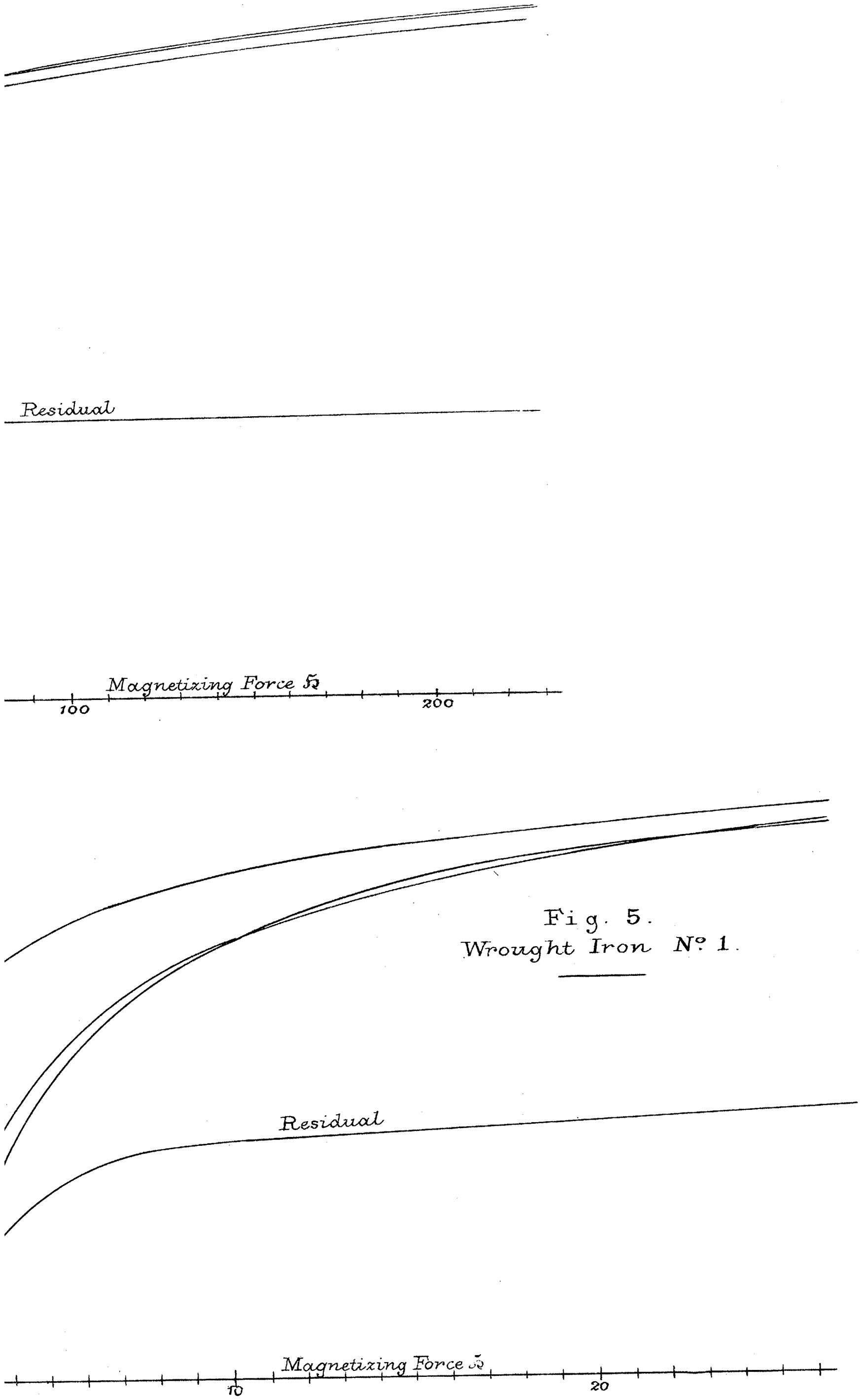


Fig. 5.
Wrought Iron No. 1.

Phil. Trans. 1885. Plate 47.

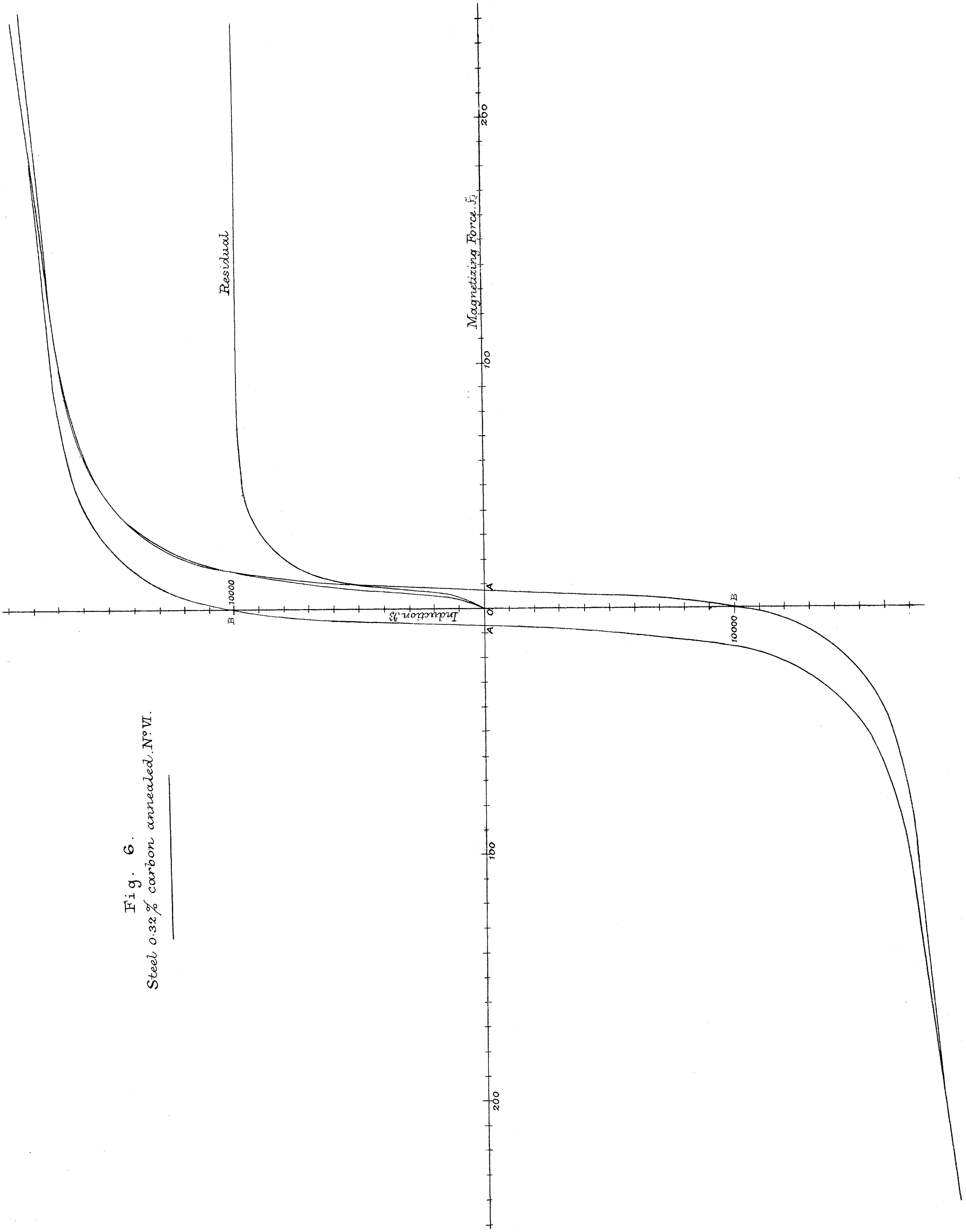


Fig. 6.
Steel 0.32% carbon annealed. N° VI.

West Newman & C° lith.

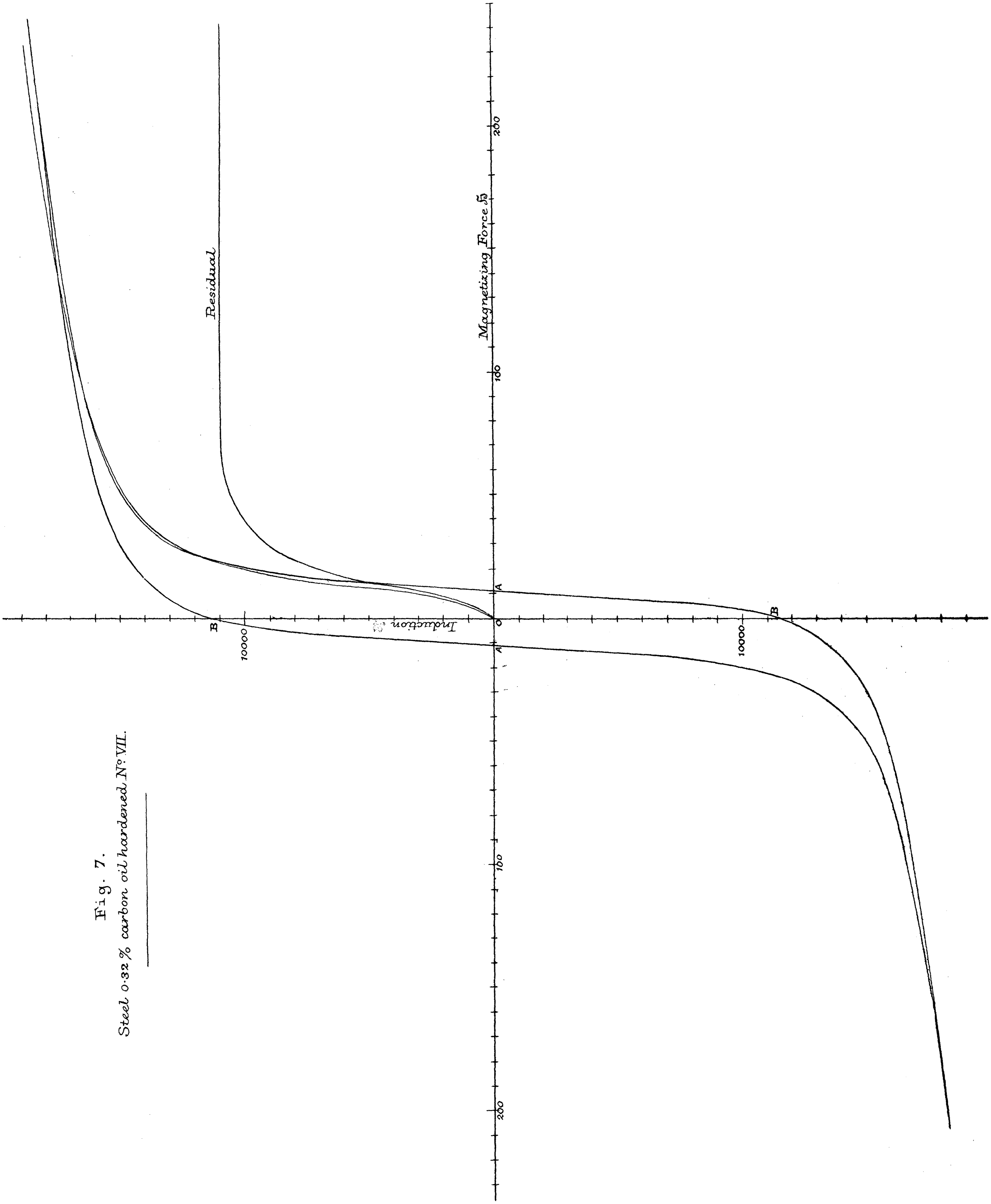


Fig. 7.
Steel 0·32 % carbon oil hardened No. VII.

West Newman & Co. lith.

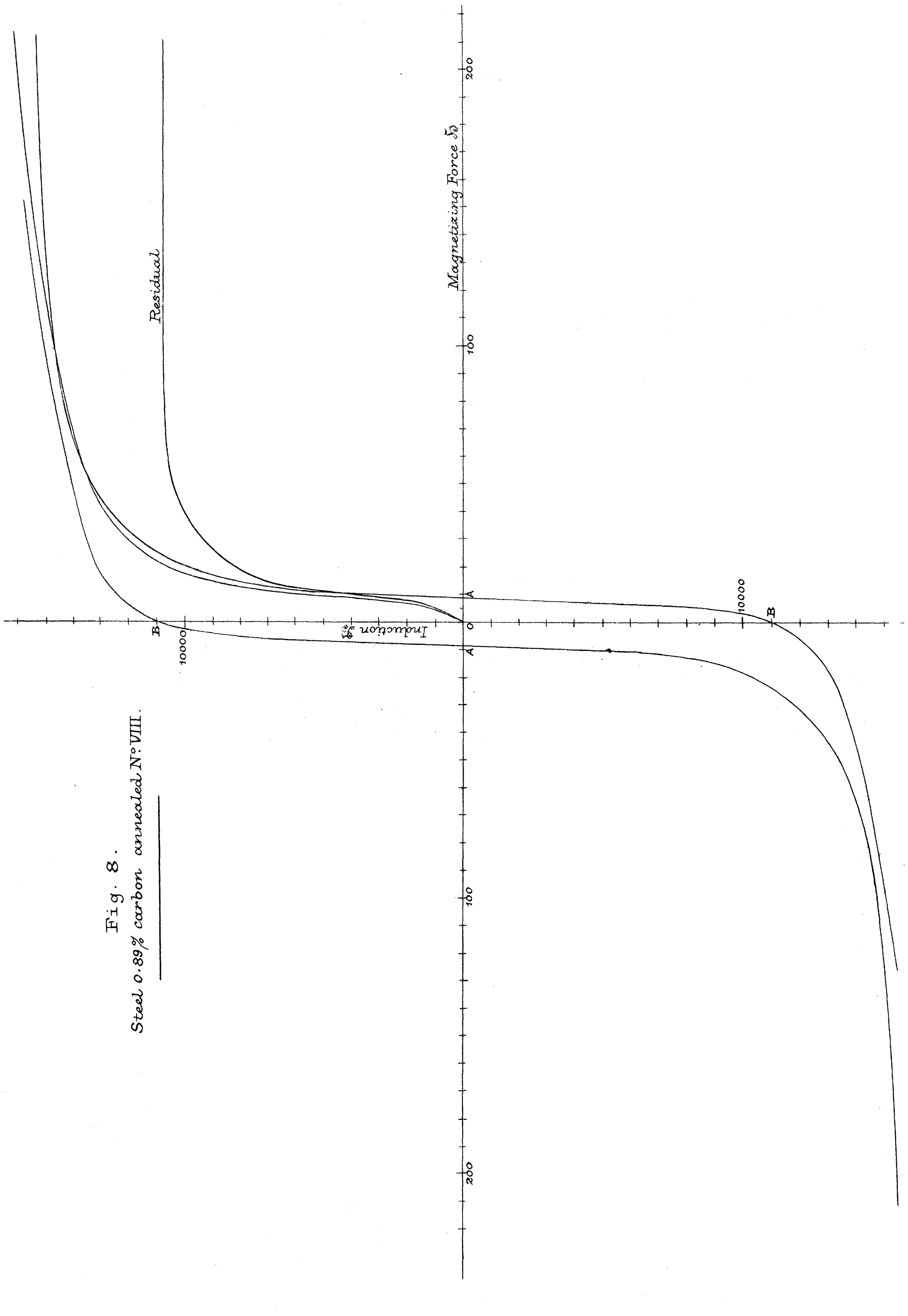


Fig. 8.
Steel 0·89% carbon annealed No. VIII.

West Newman & Co. lith.

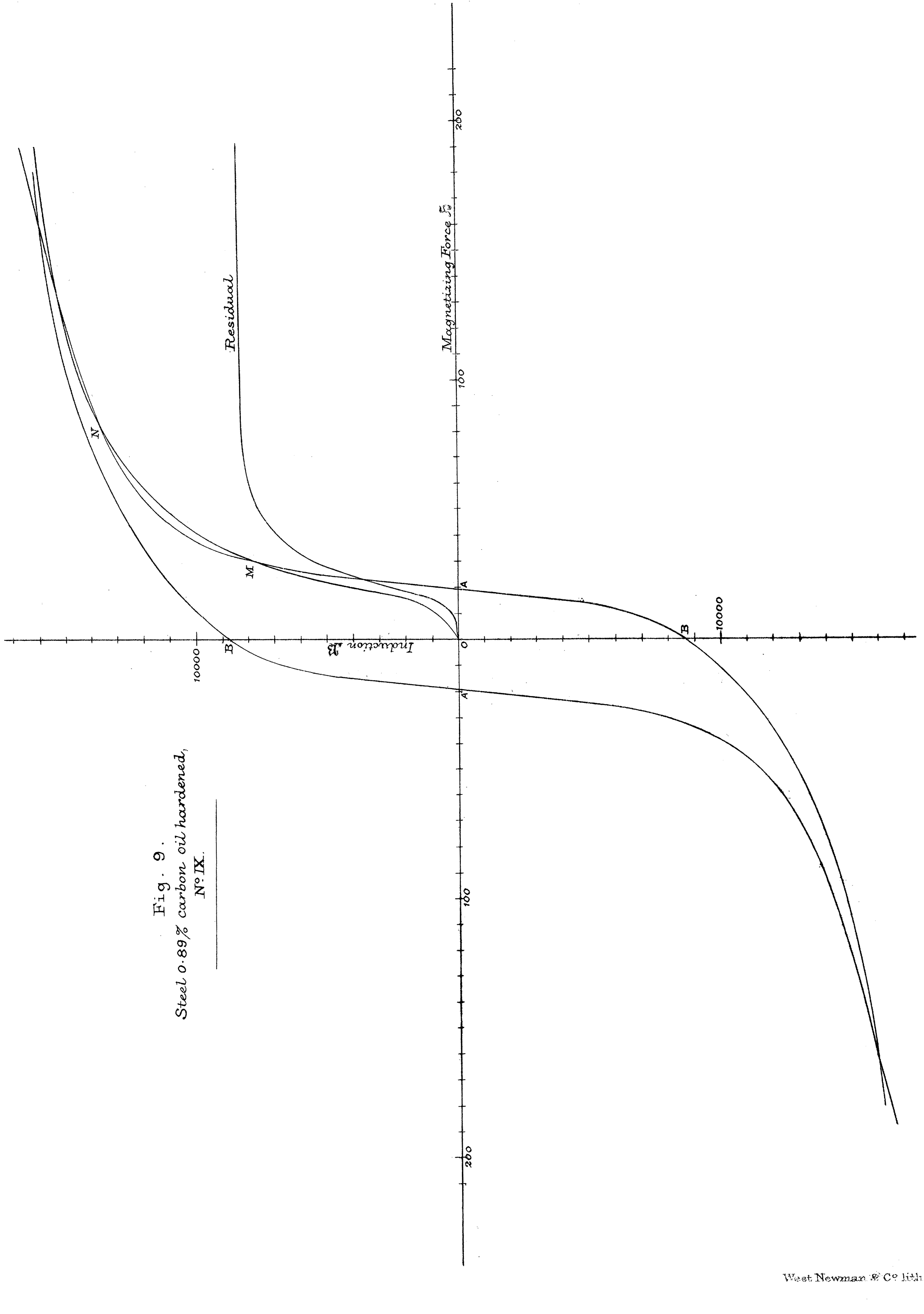


Fig. 9.
Steel 0·89% carbon oil hardened,
N° IX.

West Newman & C° lith.

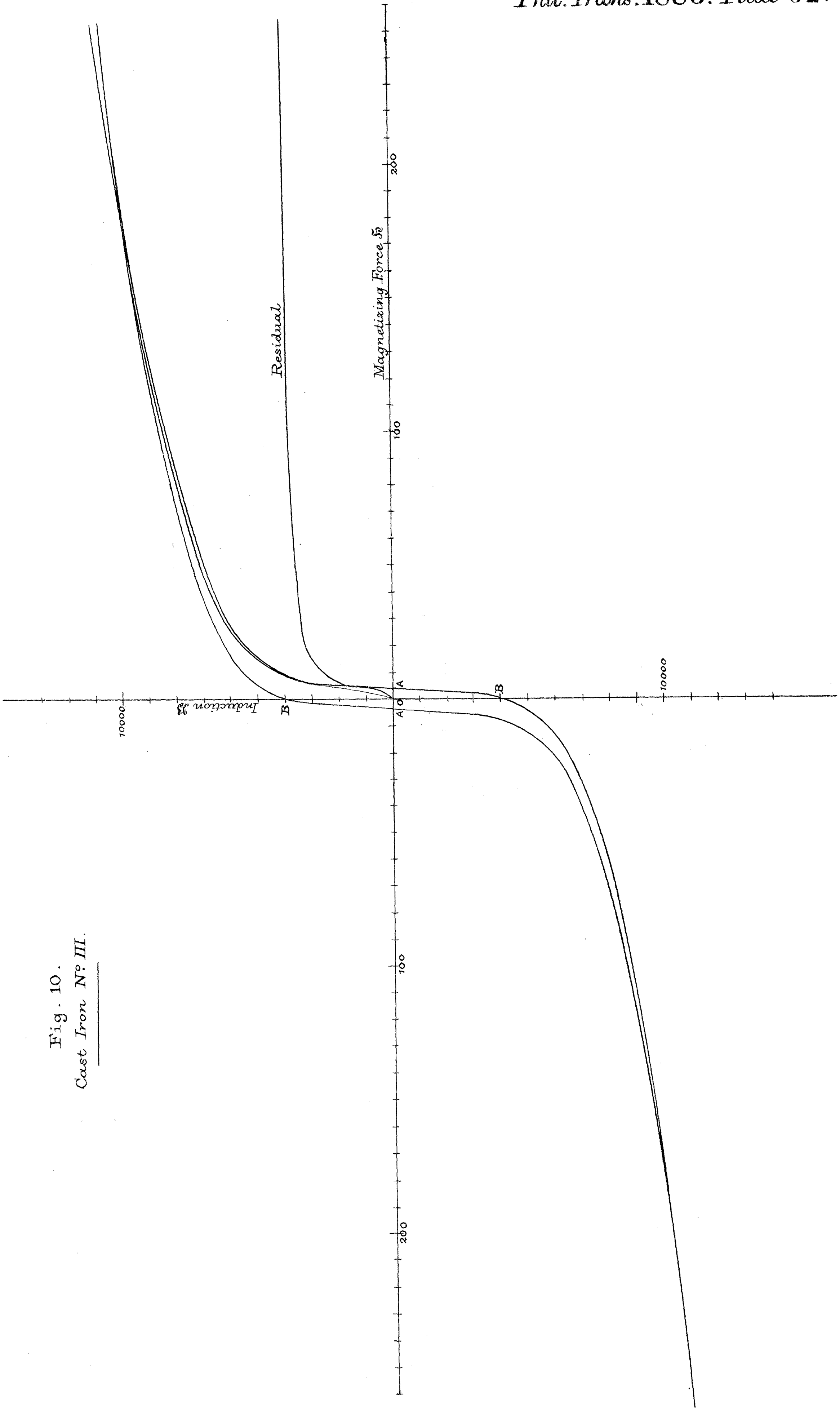


Fig. 10.
Cast Iron No. III.

West Newman & Co. lith.

Fig. 4.
Wrought Iron No. 1.

Fig. 5.
Wrought Iron No. 1.

West, Newman & Co. lith.